C.U.SHAH UNIVERSITY
Winter Examination-2018

Subject Name: Mathematics - II

Subject Code: 4SC02MAT1   **Branch: B.Sc. (All)**

Semester: 2   **Date: 31/10/2018**   **Time: 02:30 To 05:30**   **Marks: 70**

Instructions:
    (1) Use of Programmable calculator & any other electronic instrument is prohibited.
    (2) Instructions written on main answer book are strictly to be obeyed.
    (3) Draw neat diagrams and figures (if necessary) at right places.
    (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a)   Find principal value of $\log\left(1-i\sqrt{3}\right)$.   **(02)**

b)   Solve: $(D^3 - 1)y = 0$ **(02)**

c)   Find solution of $e^{4z} = i$ **(02)**

d)   Find real and imaginary part of $\sin z$.   **(02)**

e) $e^{2\pi i} =$ _____.   **(01)**
    (a) 0   (b) 1   (c) -1   (d) i

f)   Write the equation of Ellipsoid.   **(01)**

g)   True/ False: If P.I. $= 0$ then the general solution of ordinary differential equation is same as the complementary function.   **(01)**

h)   The polar form of $z = \dfrac{1+i}{\sqrt{2}}$ is _____.   **(01)**

    (a) $e^{\frac{3\pi}{4}i}$ (b) 1   (c) $e^{\frac{\pi}{4}i}$ (d) $e^{\frac{-\pi}{4}i}$

i)   The equation $2\left(x^2 + y^2 + z^2\right) - 2xy + 2yz + 2zx = 3a^2$ represents a   **(01)**
    (a) cone   (b) sphere   (c) right circular cylinder   (d) pair of planes

j)   The order of the differential equation $\left(\dfrac{d^2 y}{dx^2}\right)^{2/3} = \left[y + 5\left(\dfrac{dy}{dx}\right)\right]^{\frac{1}{3}}$ is   **(01)**

    (a) 1   (b) $\dfrac{1}{2}$ (c) $\dfrac{2}{3}$ (d) 2

Attempt any four questions from Q-2 to Q-8

Q-2   **Attempt all questions**   **(14)**
    a)   State and prove De-moivre's theorem   **(07)**

b) Find the value of $(1+i)^{40} + (1-i)^{40}$. (04)

c) Simplify: $\dfrac{(\cos 5\theta + i\sin 5\theta)^{-\frac{1}{5}}(\cos 4\theta - i\sin 4\theta)^{\frac{3}{2}}}{(\cos 3\theta - i\sin 3\theta)^{3}(\cos 2\theta + i\sin 2\theta)^{-7}}$ (03)

Q-3 Attempt all questions (14)

a) Prove that $\tan 6\theta = \dfrac{6\tan\theta - 20\tan^{3}\theta + 6\tan^{5}\theta}{1 - 15\tan^{2}\theta + 15\tan^{4}\theta - \tan^{6}\theta}$ (05)

b) Find the roots of the equation $z^{6} - 1 = 0$. (05)

c) Solve: $\left(D^{2} - 3D + 2\right)y = \sin x \cdot \sin 2x$ (04)

Q-4 Attempt all questions (14)

a) Using De-moivre's theorem solve $z^{7} + z^{4} + z^{3} + 1 = 0$. (07)

b) Solve: $x^{2}y'' - xy' + 2y = x\log x$ (07)

Q-5 Attempt all questions (14)

a) Prove that $\cosh^{-1}(z) = \log(z + \sqrt{z^{2} - 1})$. (05)

b) Find the principal value of $(-i)^{i}$. (05)

c) Find the real and imaginary part of $\tanh z$. (04)

Q-6 Attempt all questions (14)

a) If $z^{2}\sin^{2}\theta - z\sin\theta + 1 = 0$ then prove that $\alpha^{n} + \beta^{n} = 2\cos\left(\dfrac{n\pi}{3}\right)\cos ec^{n}\theta$ (07)

b) Solve: $(3x + 2)^{2}y'' + 3(3x + 2)y' - 36y = 3x^{2} + 4x + 1$. (07)

Q-7 Attempt all questions (14)

a) Identify, describe and sketch the surface $16x^{2} + 36y^{2} + 9z^{2} = 144$. (05)

b) Find the enveloping cone of the sphere $x^{2} + y^{2} + z^{2} - 2x + 4z = 1$ with its vertex at $(1, 1, 1)$. (05)

c) Solve :$\left(D^{2} - 1\right)y = \cos x \cdot \sinh x$ (04)

Q-8 Attempt all questions (14)

a) The vertex of cone is (a, b, c) and the $yz - plane$ cuts it in the curve $F(y, z) = 0,\ x = 0$, show that $xz - plane$ cuts it in the curve $y = 0, F\left[\dfrac{bx}{x-b}, \dfrac{cx-az}{x-a}\right] = 0$. (05)

b) Find equation of cylinder whose generators are parallel to $\dfrac{x}{3} = \dfrac{y}{2} = \dfrac{z}{1}$ and guiding curve $x^{2} + y^{2} + z^{2} = 9$. (05)

c) Solve: $\left(D^{2} - 6D + 5\right)y = e^{3x}$ (04)